1st Brazilian Workshop on Interior Point Methods

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The advantages of interior point methods for decomposition techniques

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FAPESP CNPq

Outline

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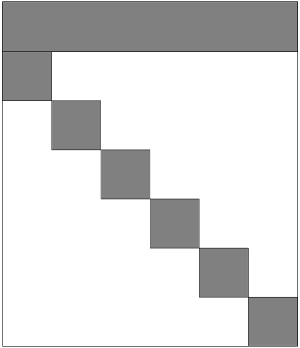

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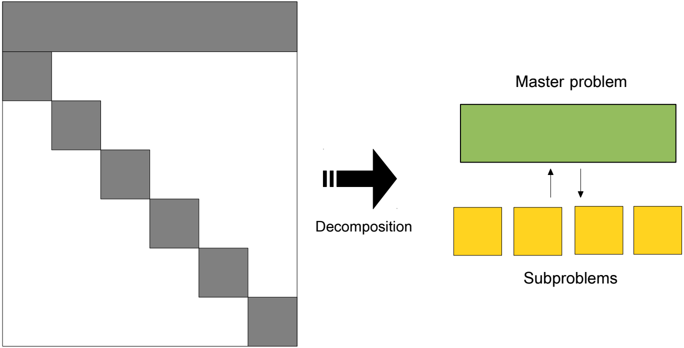

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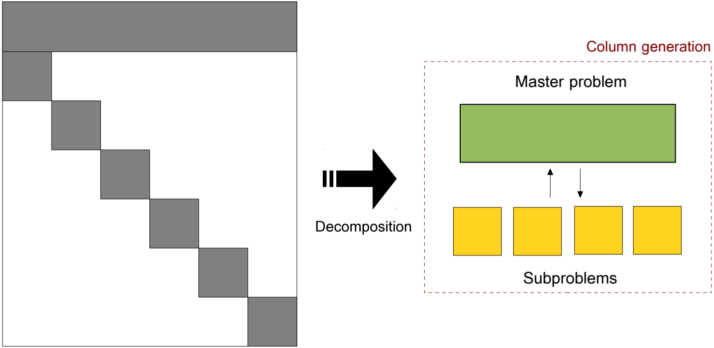

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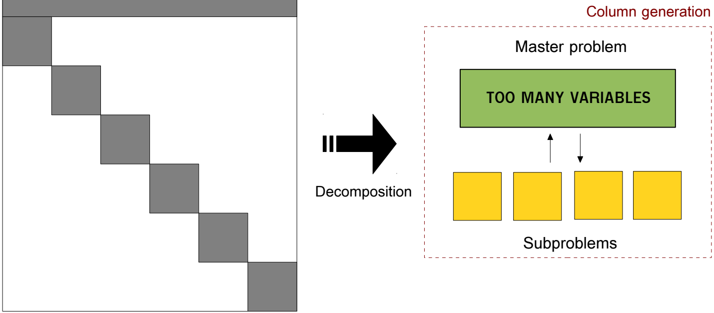

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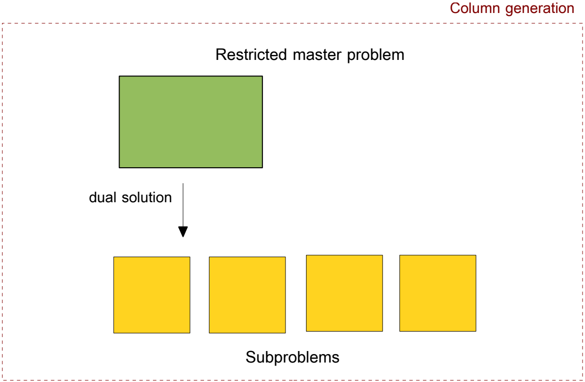

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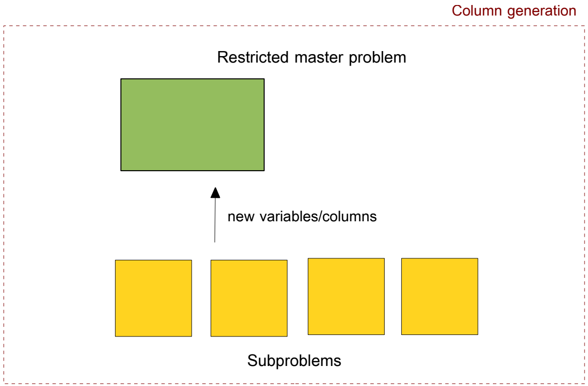

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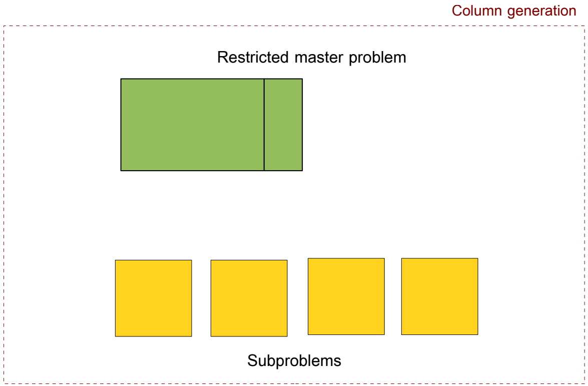

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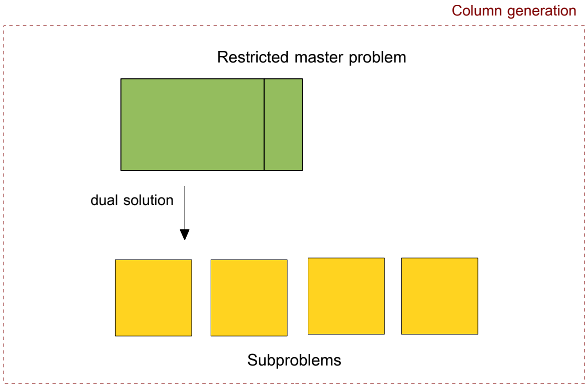

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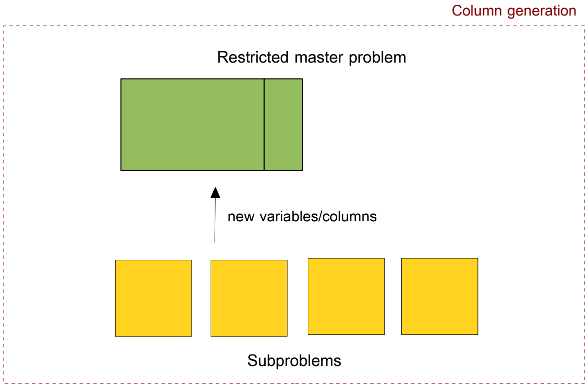

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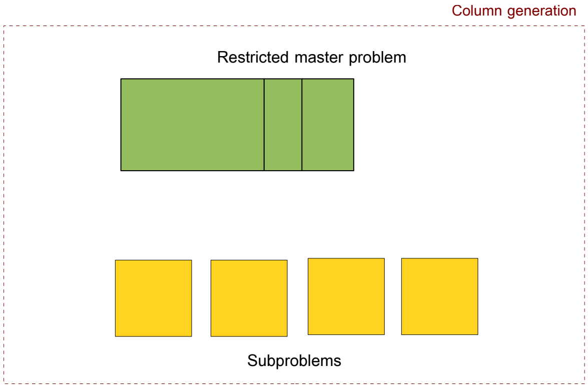

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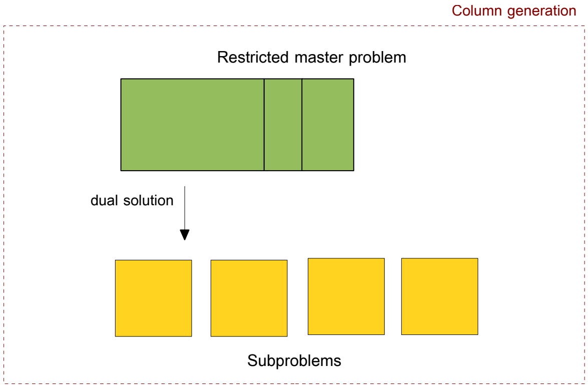

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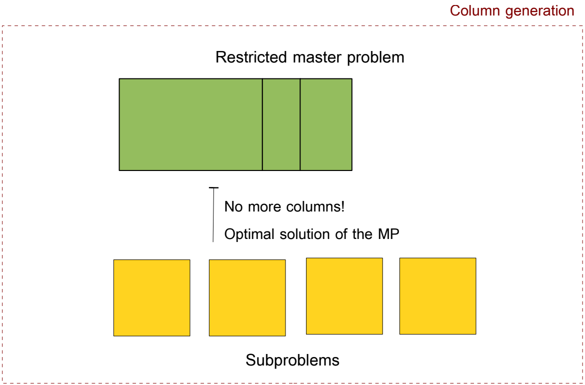

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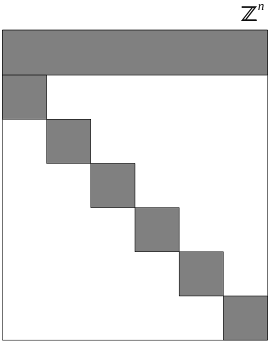

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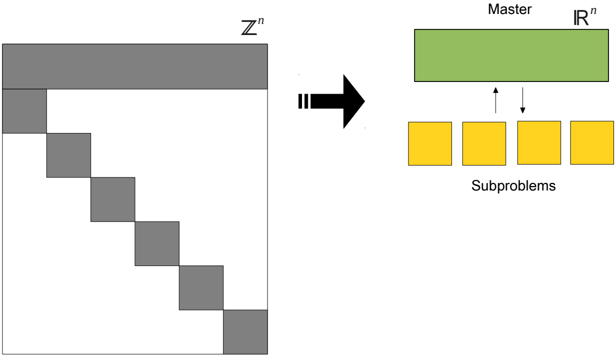

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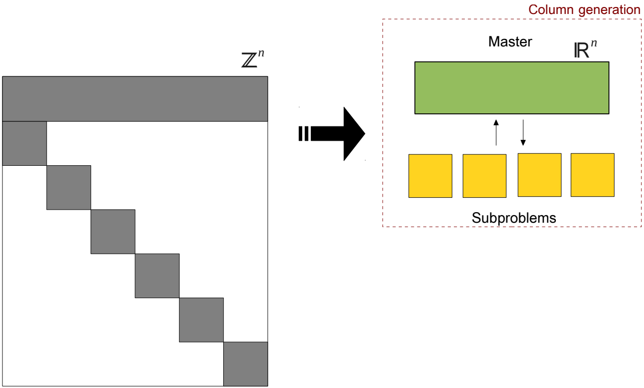

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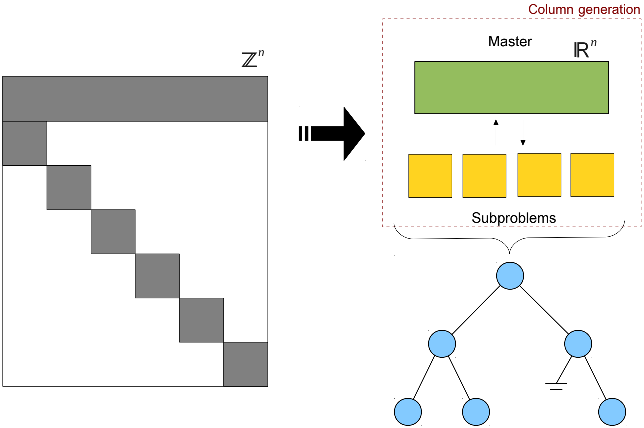

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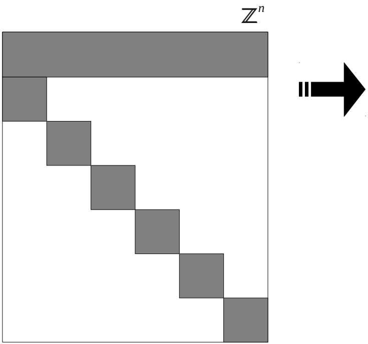

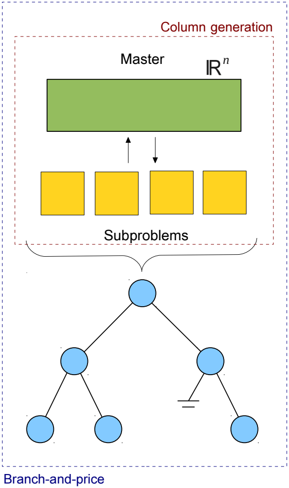

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- This way, it is important to use an efficient LP algorithm;
- We should exploit also additional advantages offered by the algorithm.

Column generation

- We are interested in solving a linear programming problem, called the Master Problem (MP):

$$\begin{aligned} z^\star := \min \quad & \sum_{j \in N} c_j \lambda_j, \\ \text{s.t.} \quad & \sum_{j \in N} a_j \lambda_j = b, \\ & \lambda_j \geq 0, \qquad \forall j \in N. \end{aligned}$$

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- We know how to generate them!

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- Otherwise, an optimal solution of the MP was found!

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- Slow convergence of the method.

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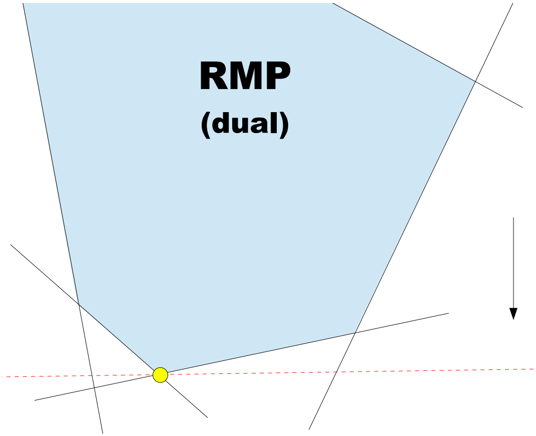

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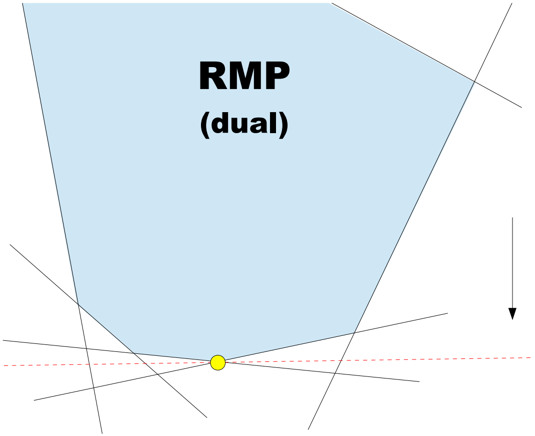

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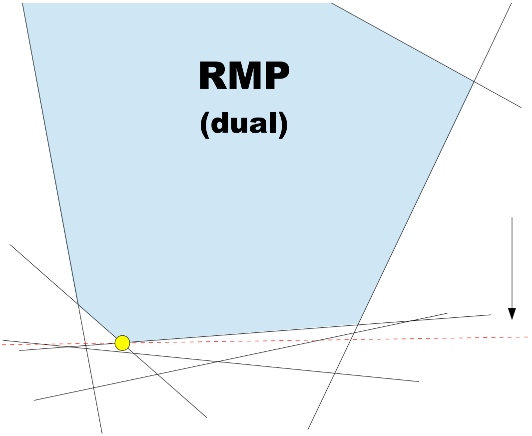

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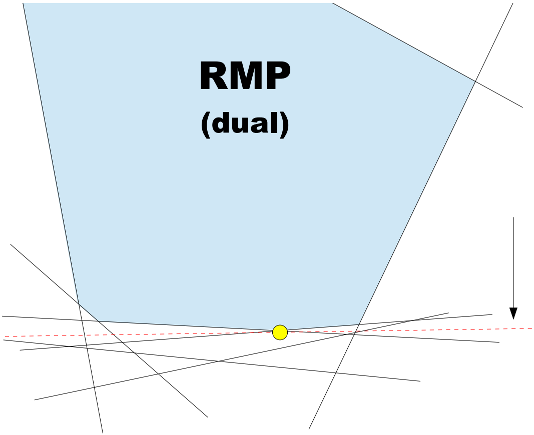
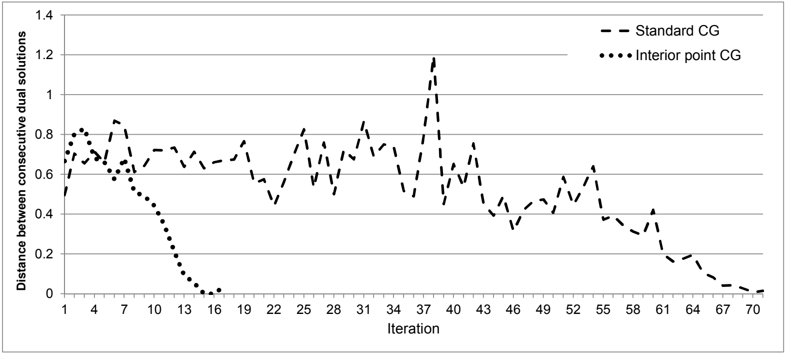


Oscillation in a real instance

$\|u^j - u^{j+1}\|_2$, for each iteration j:



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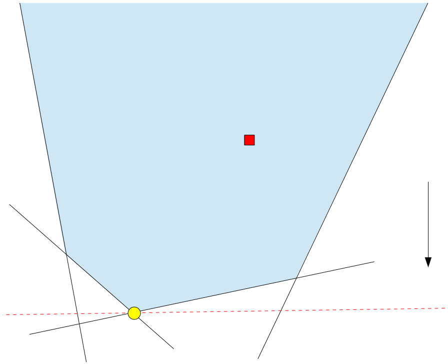

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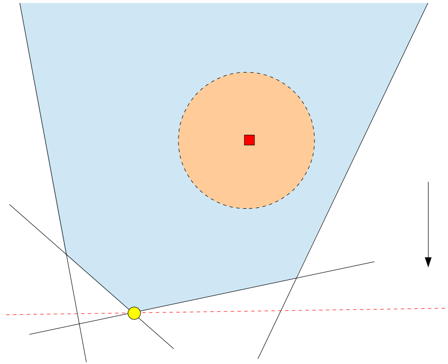

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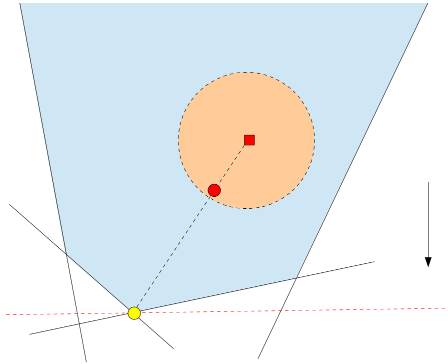

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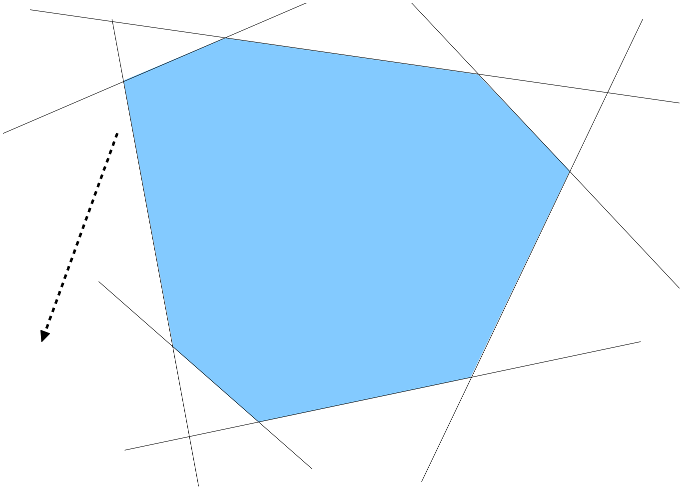
- This is straightforward: does not require any changes in the RMP nor parameter adjustments;
- Interior point methods will provided naturally stable solutions.

Interior point method



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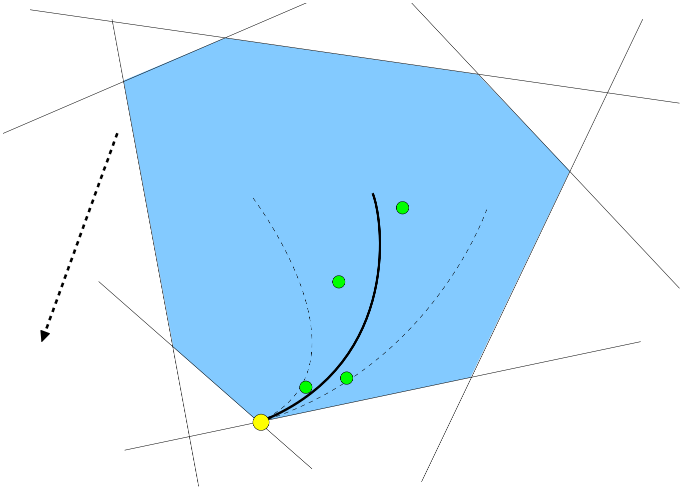

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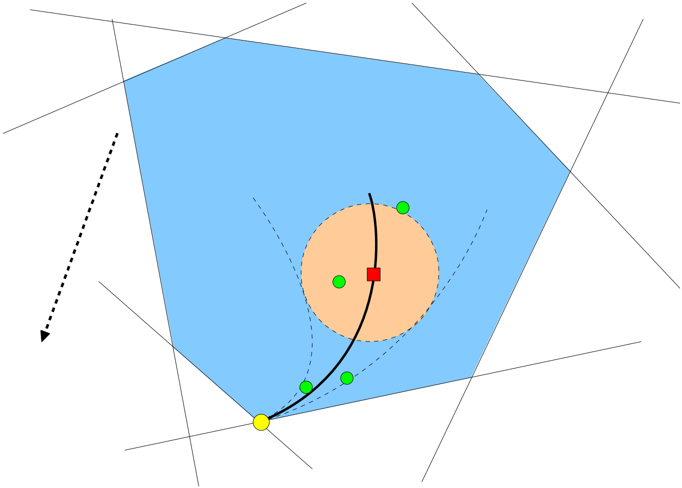

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- The column corresponds to a deeper cut in the dual space.

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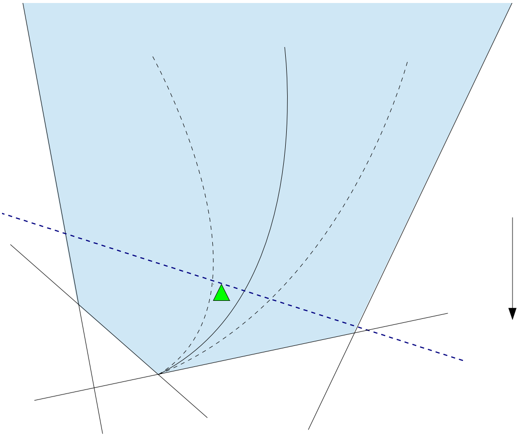

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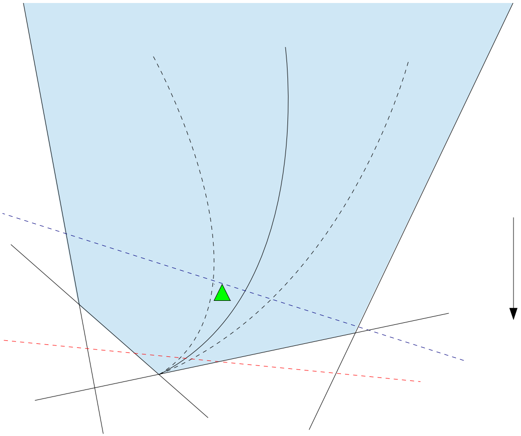

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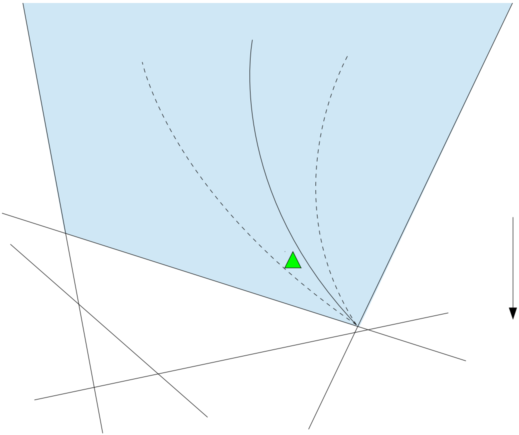

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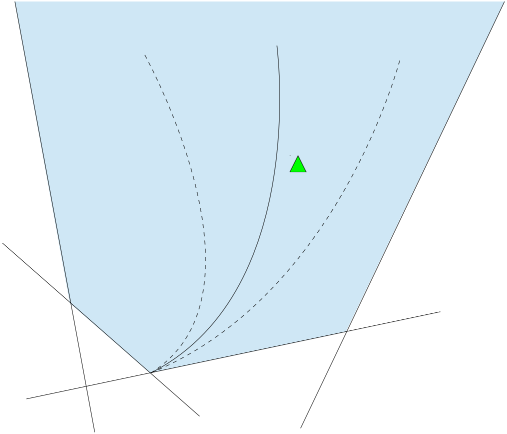

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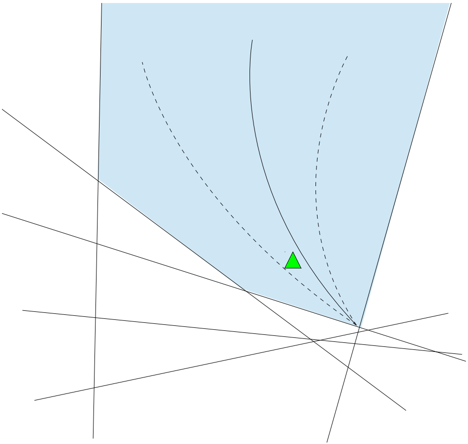

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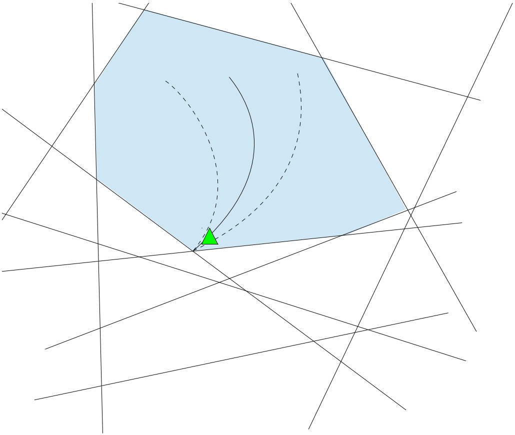

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3. while (gap $> \delta$) do
4. find a well-centered ε-optimal solution $(\tilde{\lambda}, \tilde{u})$ of the RMP;
5. UB $= \min\{\text{UB}, \tilde{z}_{RMP}\}$;
6. call the oracle with the query point $\tilde{u}$;
7. LB $= \max\{\text{LB}, \kappa \tilde{z}_{SP} + b^T \tilde{u}\}$;
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- Source-code examples are provided for 6 different applications:
  - Cutting stock problem;
  - Vehicle routing problem;
  - Capacitated lot sizing;
  - Multiple kernel learning;
  - Two-stage stochastic programming;
  - Multicommodity network flow.

European Journal of Operational Research 224 (2013) 41–51

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Discrete Optimization

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[a] *School of Mathematics, The University of Edinburgh, James Clerk Maxwell Building, The King's Buildings, Mayfield Road, Edinburgh EH9 3JZ, United Kingdom*
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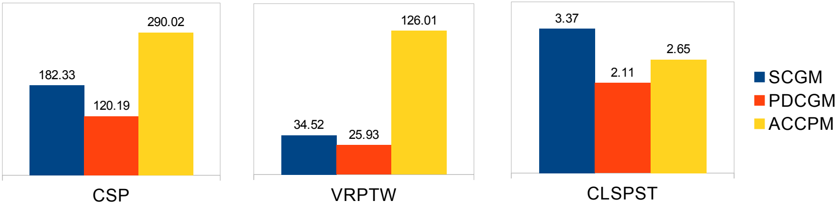
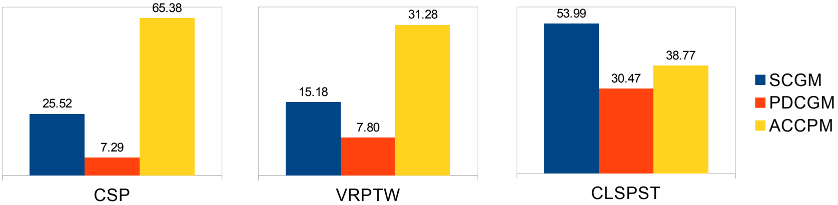
  - Cutting stock problem (CSP);
  - Vehicle routing problem with time windows (VRPTW);
  - Capacitated lot sizing with setup times (CLSPST).

Number of iterations



Relative to PDCGM	CSP	VRPTW	CLSPST
SCGM	1.52	1.33	1.60
ACCPM	2.41	4.86	1.26

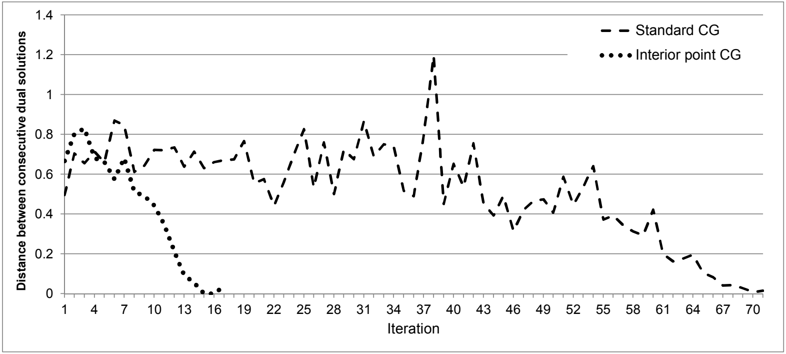
CPU time (s)



Relative to PDCGM	CSP	VRPTW	CLSPST
SCGM	3.50	1.95	1.26
ACCPM	8.97	4.01	1.27

Oscillation in a VRPTW instance (Solomon C207)

$\|u^j - u^{j+1}\|_2$, for each iteration j:



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Two-stage stochastic programming problem (TSSP)

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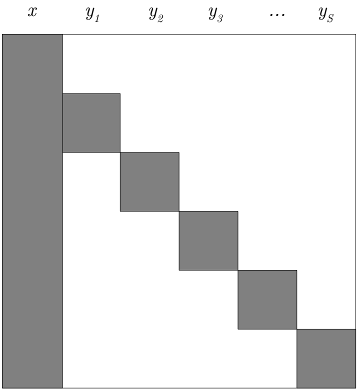
- Model many real-life situations taking uncertainty into account;
- Two interconnected problems: first-stage and recourse;
- Deterministic equivalent problem (DEP):

$$
\begin{aligned}
\min_{x,y} \quad & c^T x + \sum_{i \in \mathcal{S}} p_i q_i^T y_i, & \\
\text{s.t.} \quad & Ax = b, & \\
& T_i x + W_i y_i = h_i, & \forall i \in \mathcal{S}, \\
& x \geq 0, & \\
& y_i \geq 0, & \forall i \in \mathcal{S}.
\end{aligned}
$$

Special structure of the model



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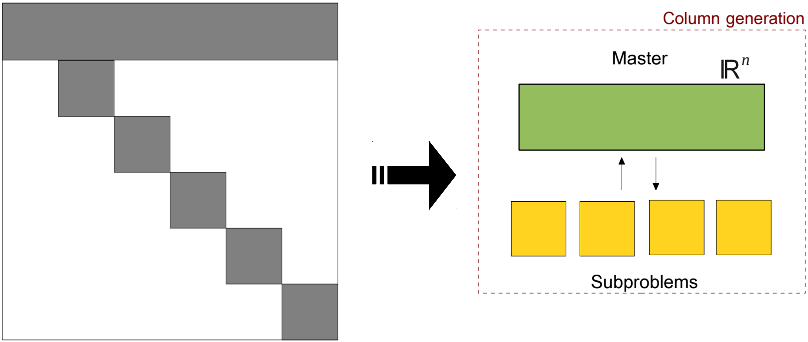

TSSP: Computational experiments

- TSSP instances that have been widely used in literature (Ariyawansa and Felt, 2004; Holmes, 1995);
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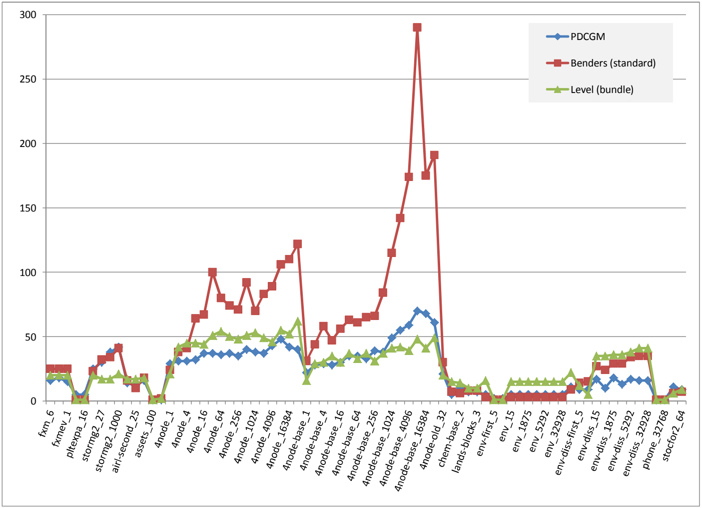
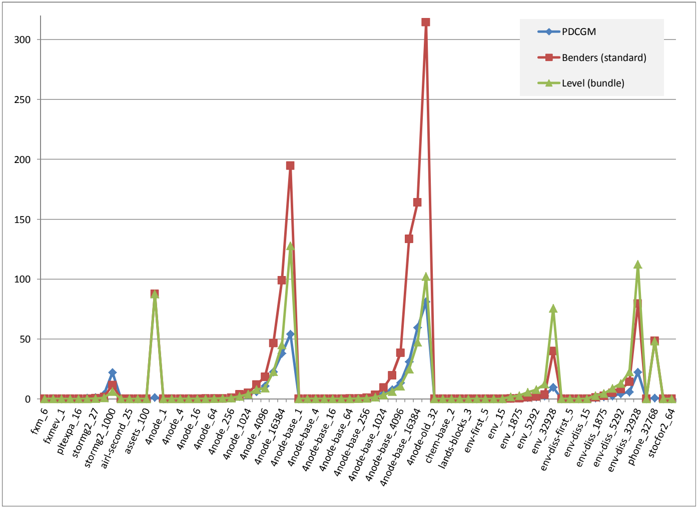
- We have taken the results of both methods from Zverovich et al. (2012), which used a Core i5 2.4 GHz CPU and 6 GB of memory.

TSSP: Number of iterations



TSSP: CPU Time (seconds)



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journal homepage: www.elsevier.com/locate/caor

Using the primal-dual interior point algorithm within the branch-price-and-cut method

Pedro Munari [a,*,1], Jacek Gondzio [b]

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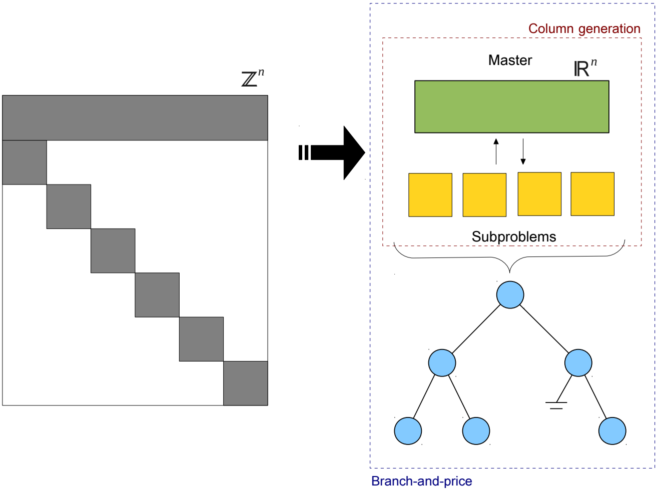
Interior point branch-price-and-cut method:

- Primal-dual interior point method;
- Well-centered dual solutions to generate columns and valid inequalities;
- Early termination;
- Vehicle routing problem with time windows (VRPTW);

Large-scale discrete problems



Interior point branch-price-and-cut (IPBPC)

- Vast majority of branch-price-and-cut methods are based on optimal solutions obtained with the simplex method;

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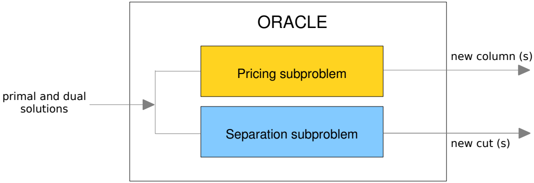
- Vast majority of branch-price-and-cut methods are based on optimal solutions obtained with the simplex method;
- Change of strategy!
- It is not just replacing a simplex-type method.
- Rethink every piece of a standard BPC: column generation, valid inequalities, branching, ...

Interior point branch-price-and-cut (IPBPC)

- Primal-dual column and cut generation:
  - We modify the Oracle: two types of subproblems;



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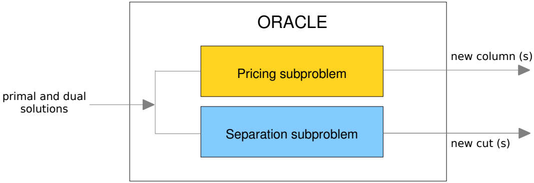

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  - Stop CG with a loose tolerance (e.g. 10^{-3}) and branch!

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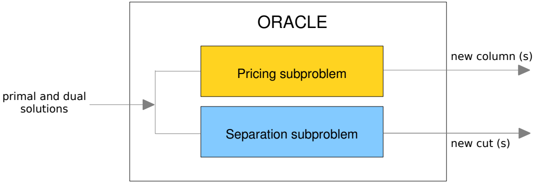
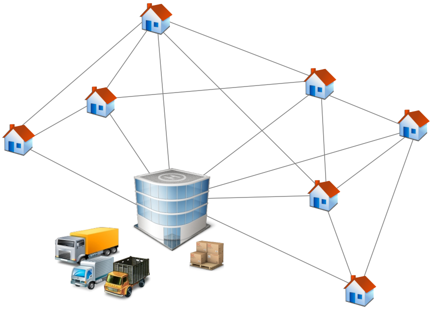
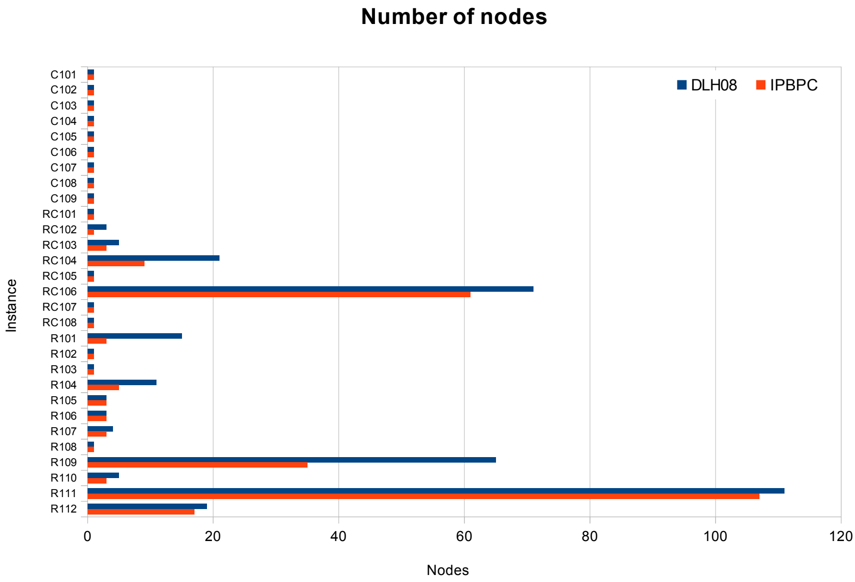


- Early branching:
  - Stop CG with a loose tolerance (e.g. 10^{-3}) and branch!
- In all cases: we use a well-centered suboptimal solution.

Vehicle Routing Problem with Time Windows (VRPTW)



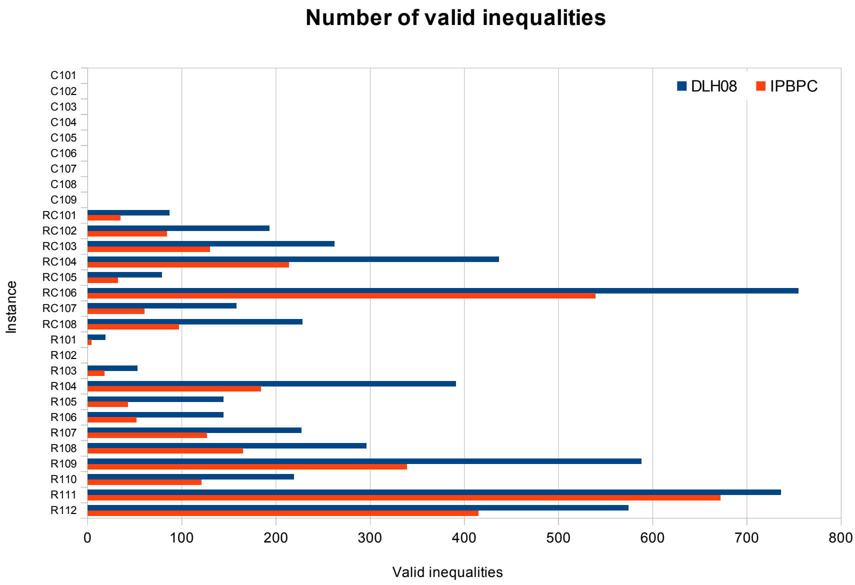
- Interior point branch-price-and-cut (IPBPC);
- The IPBPC performance was compared to the best results that were available in the literature for a *simplex-based* BPC:
  - DLH08: *Desaulniers, Lessard and Hadjar (2008), Transp. Science.*


Number of nodes
DLH08
IPBPC
Instance
C101
C102
C103
C104
C105
C106
C107
C108
C109
RC101
RC102
RC103
RC104
RC105
RC106
RC107
RC108
R101
R102
R103
R104
R105
R106
R107
R108
R109
R110
R111
R112
0
20
40
60
80
100
120
Nodes

Comparing to a simplex-based BPC

Number of nodes

	DLH08	**IPBPC**	**Ratio**
C1	9	9	1.00
RC1	104	78	1.33
R1	239	182	1.31
	352	269	1.31

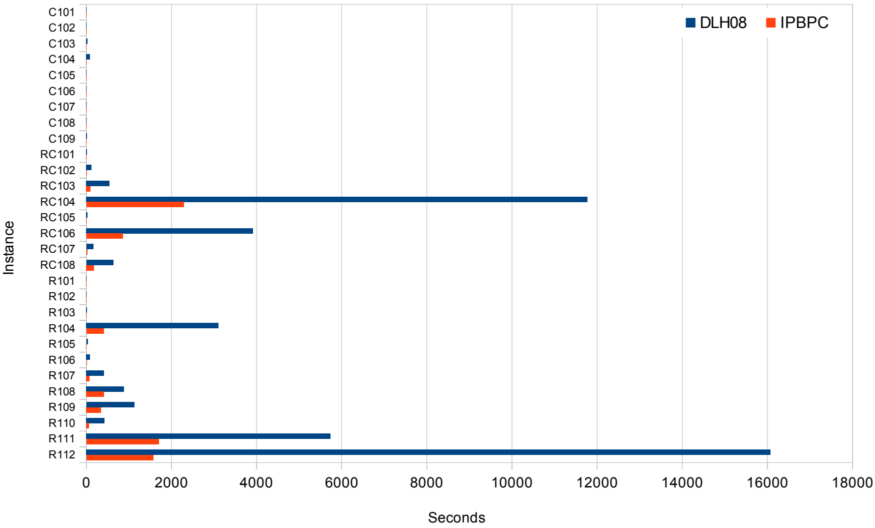

Number of valid inequalities
DLH08
IPBPC
Instance
C101
C102
C103
C104
C105
C106
C107
C108
C109
RC101
RC102
RC103
RC104
RC105
RC106
RC107
RC108
R101
R102
R103
R104
R105
R106
R107
R108
R109
R110
R111
R112
0
100
200
300
400
500
600
700
800
Valid inequalities

Comparing to a simplex-based BPC

Number of valid inequalities

	DLH08	IPBPC	Ratio
C1	0	0	1.00
RC1	2199	1191	1.85
R1	3391	2140	1.58
	5590	3331	1.68

CPU time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C101
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RC105
RC106
RC107
RC108
R101
R102
R103
R104
R105
R106
R107
R108
R109
R110
R111
R112
0
2000
4000
6000
8000
10000
12000
14000
16000
18000
Seconds

Comparing to a simplex-based BPC

CPU time (sec)

	DLH08	**IPBPC**	**Ratio**
C1	158	28	5.69
RC1	17198	3472	4.95
R1	27928	4621	6.04
	45284	8121	5.58

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- Does not require changes to the RMP nor parameter tunning;
- Computational experiments indicate that this approach is successful in different types applications;
- Reductions in the number of iterations and CPU time when compared to standard column generation, ACCPM, bundle methods.

Thank you!

Questions?

- Acknowledgments

FAPESP CNPq

- PDCGM webpage:

  `http://www.maths.ed.ac.uk/~gondzio/software/pdcgm.html`

- munari@dep.ufscar.br